

Magnetic properties of superconducting multifilamentary tapes in perpendicular field. I: Model and vertical stacks

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Current and field profiles, and magnetization and AC losses are calculated for an array of infinitely long superconducting tapes arranged vertically in a perpendicularly applied magnetic field. Calculations are based on the critical state model. The finite thickness of the tapes and the effects of demagnetizing fields are considered. The influence of the magnetic coupling of the filaments in the magnetic properties of the arrays are studied. The general model can be applied to an arbitrary arrangement of tapes as long as there is reflection symmetry with respect to the vertical central plane.

I. INTRODUCTION

The magnetic behavior of hard superconductors depends not only on their intrinsic properties, such as the critical-current density J_c , but on their geometry. For example, the magnetization and ac losses for a superconducting thin film are very different depending on whether the magnetic field is applied perpendicular or parallel to the thin dimension. Recent advances in the modeling of magnetic properties of superconductors (SCs) have allowed to understand their behavior in a large variety of geometries. The most common framework used to address this problem has been the critical state-model [1], that assumes that currents circulating in the SCs flow with a constant density J_c , later extended to currents depending only upon the local magnetic field H_i [2]. The original model was solved in the parallel geometry, that is for applied fields H_a along an infinite dimension for slabs and cylinders [1–3], because in these geometries the problem of the demagnetizing effects was not present. A further step was presented when the CSM was extended to the case of very thin strips [4–6] and disks [7–9], for which important demagnetizing fields were involved. More recently, the more general case of critical state in samples with finite thickness, such as strips [10] and cylinders [11–13] was solved by numerical models.

However, there is a particular arrangement that has not been solved so far in spite of its practical importance. It is the case of a set of superconductors with finite thickness arranged in an array form. The importance of this case is not only academic but practical, since this geometry is often a good system for modeling the filamentary structure of actual superconducting tapes. The recent advances in the technology of superconducting Ag/Bi2223 multifilamentary tapes increase the importance of having a theory describing the magnetic properties of such arrays.

Although the problem of studying the current distribution and magnetization in array of superconducting tapes has not been systematically solved, there have been significant works offering partial solutions. Fabbricatore et al [14] presented a comprehensive analysis of the Meissner

state in arrays of strip lines arranged vertically (z stack of strips), horizontally (x arrays) and in the form of a matrix (xz array) and compared their results with actual measurements on multifilamentary tapes. Their numerical procedure, however, was not adequate to study the more general case of bulk current penetration. Mawatari [15] studied not only the Meissner state but also the critical state for the case of an infinite set of periodically arranged superconducting-strip lines, in the limit that the strip lines were infinitely thin. Mawatari and Clem [16] studied the penetration of magnetic flux into current-carrying (infinitely thin) strips lines with slits in the absence of applied magnetic field. All the existing models assume either arrays of infinitely thin strips in the critical state or arrays of strips with finite thickness but only in the Meissner state. The only exception we know is the recent work by Tebano et al [17] in which preliminary results on the current penetration and magnetization were calculated for some realistic arrays based on the procedure developed by Brandt [10].

A key issue in the study of superconducting tapes is how currents circulate within filaments. There are two important cases concerning this point, depending on whether current in each filament is restricted to go and return through the same filament or if instead there is no such restriction. In principle, both cases have practical and theoretical interest, as we discuss in the second paper of this series [18]. The magnetic response and AC losses will strongly depend on this factor so it is imperative to study both cases separately and compare them.

In this series of papers we study the current and field penetrations, magnetization and AC losses of arrays of superconducting strips of finite thickness. In the first paper we present the model and its application to the case of an array of a finite number of infinitely long strips of finite thickness arranged vertically (z stack of finite strips) with a perpendicular applied field. This geometry is studied first because it is independent of the connection type, since because of the system symmetry, current always go and return through each strip. In the second paper we will study the cases of horizontal (x) arrays and matrices (xz array), for which different behaviors

arise depending on the connections. In all cases, we will concentrate our study in tapes composed of strips with high aspect ratio since this is the case most often met in practice, although our model is applicable to arrays of strips with arbitrary thickness. However, this is different from the approximation of considering infinitely thin strips as in [15], since we take into account the different current penetration across the superconductor thickness.

The present paper is structured as follows. In section II we introduce the calculation model. Current and field profiles are calculated and discussed in Section III. The results of magnetization and ac losses are discussed in Section V and VI, respectively. In Section VII we present the main conclusions of this work. In two appendices we present analytical expressions for both the full penetration field of the arrays and the inductances used in the model calculations.

II. MODEL

The model we present here is suitable for any superconductor geometry with translational symmetry along the y axis and mirror symmetry to the vertical plane. However, we will focus on z -stacks, x -arrays, and xz -matrices made up of identical strips infinitely long in the y direction. The horizontal and vertical strips dimensions are $2a$ and $2b$, respectively. We consider a uniform applied field H_a in the z direction. Results for z -stacks are discussed in the present paper, whereas those for x -arrays and xz -matrices are presented in the second paper of this series [18].

Our numerical model is based on minimizing the magnetic energy of the current distribution after each applied field variation. The model assumes there is no equilibrium magnetization in the superconductor and, in the present series of papers, we further assume there is no field dependence of J_c for simplicity. The energy and flux minimization in the critical state was previously discussed by Badia [19] and Chaddah et al [20,21]. The details of the general numerical model can be found in [22,13,23,24]. The approach has been successfully applied to describe the experimental features observed in the initial magnetization slope [22] as well as the whole magnetization loop and levitation force [13,23] of superconducting cylinders. We now outline the main characteristics of the model.

For cylindrical geometry [22,13,23], the superconducting region was divided into a certain number of elements of the shape of rings with rectangular cross-section, which formed circular closed circuits. For a given current distribution in the cylinder, the energy variation of setting a new current in a certain element was calculated. The minimum energy variation method consists in, given an applied field H_a , looking for the circuit which lowers the most the magnetic energy and set there a new current of magnitude J_c , and then repeat the procedure until set-

ting any new current does not reduce the energy. The method as explained above is used to find the initial magnetization curve $M_i(H_a)$. Provided that J_c is field independent, the reverse curve can be found using that [9]

$$M_{\text{rev}}(H_a) = M_i(H_m) - 2M_i((H_m - H_a)/2), \quad (1)$$

and the returning curve using

$$M_{\text{ret}}(H_a) = -M_{\text{rev}}(-H_a), \quad (2)$$

where H_m is the maximum applied field in the loop.

In the present case, each strip is divided into a set of $2n_x \times 2n_z$ elements with cross-section $(\Delta x)(\Delta z)$ and an infinite length in y direction, as shown in Fig. 1. The dimensions of each element are $\Delta x = a/n_x$ and $\Delta y = b/n_y$. We consider that the current density is uniform within the elements and flows through the whole element section and not only through a linear circuit, as in [22,13,23].

When the distribution and orientation of the strips in the set are symmetrical to the zy plane, as is the case for a z -stack, in the presence of a uniform applied field the induced current front will also be symmetrical to the zy plane. Thus, we can consider that the pair of elements centered at (x, z) and $(-x, z)$ form circuits that are closed at infinity. This grouping in pairs, forming closed circuits, allows for the analytical calculation of self and mutual inductances per unit length of the circuits with finite cross-section (see the Appendix B for inductances derivations and formulae). The pairs, or circuits, are labelled using the subscript i from 1 to $N = 2n_x n_z n_f$, being n_f the number of strips of the set and N the total number of elements in the $x \geq 0$ portion of the set of strips.

Once the analytical expressions for the inductances are obtained, the energy of the i circuit can be calculated as

$$E_i = \sum_{j=1}^N M_{ij} I_j I_i + 2\mu_0 H_a x_i I_i, \quad (3)$$

where the first term is the energy of the circuit owing to the presence of the current distribution in the whole superconducting region, the second term is the energy due to the uniform applied magnetic field, M_{ij} are the self and mutual inductances from Eqs. (B5)-(B7), and I_i and I_j are the total current intensity that flows through the circuits labelled as i and j , respectively. Since no internal field dependence is considered for the critical current, $|I_i| = (\Delta x)(\Delta z)J_c$. The sign of I_i is taken as positive when the current of the element at $x \geq 0$ of the pair follows the positive y axis direction and negative otherwise.

In the initial magnetization curve, after using the energy minimization procedure for a given applied field H_a to find the current profile, we can calculate the magnetization, the total magnetic field, and the magnetic field lines directly from the current distribution.

The magnetization, defined as the magnetic moment per unit volume, has only one non-zero component, M_z , which can be calculated as

$$M_z = \frac{m_z}{4abn_f} = \frac{1}{4abn_f} \sum_{i=1}^N I_i (2x_i), \quad (4)$$

where m_z is the total magnetic moment of the set of strips.

The two nonzero components of the total magnetic flux density B_z and B_x are calculated as the addition of all the closed circuit contributions, which can be calculated analytically integrating the Biot-Savart law [25].

The magnetic flux lines are calculated as in [10] using that for translational symmetry the level curves of the y component of the vector potential, for the gauge $\nabla \cdot \mathbf{A} = 0$, can be taken as the magnetic flux lines. The y component of the vector potential results from the contribution of all the elements pairs $A_{y,i}$, whose analytical expressions are the Eqs. (B2)-(B4).

We will consider the application of an external AC field $H_a = H_m \cos(\omega t)$. In this case we can calculate the imaginary part of the AC susceptibility, χ'' , defined as

$$\begin{aligned} \chi'' &= \frac{2}{\pi H_m} \int_0^\pi d\theta M_{\text{rev}}(\theta) \sin \theta \\ &= \frac{2}{\pi H_m^2} \int_{-H_m}^{H_m} dH_a M_{\text{rev}}(H_a). \end{aligned} \quad (5)$$

The energy loss W per AC cycle of amplitude H_a is related to χ'' by the expression [26,9]

$$W = \mu_0 \pi H_a^2 \chi'' \quad (6)$$

III. CURRENT PENETRATION AND FIELD LINES

The most important issue to study in the system of superconducting strips is the influence of magnetic coupling. It is well known that the superconductors tend to shield the magnetic field change not only in their interior but also in the space between two superconducting regions (as illustrated in the classical example of a hollow superconducting tube, for which the whole volume inside the tube and not only the tube walls are shielded). In our array of superconducting strips one should observe a related effect, because the current induced in the strips will tend to shield the effect of the magnetic field in their interior as well in the space between them. To study this effect we present in Fig. 2 calculations corresponding to a vertical array of three strips with $b/a = 0.1$ and different separations ($h = 2a$, $h = 0.2a$, and $h = 0.02a$, respectively). The different profiles correspond to applied fields $H_a/H_{\text{pen}} = 0.2, 0.4, 0.6, 0.8$, and 1 , where H_{pen} is the field at which the array is fully penetrated by current (the calculation of such a field is discussed in Appendix 1). The

results show unambiguously the strong influence of magnetic coupling in the case that the separation is small, as illustrated in the case for $h = 0.02a$ for which the current profiles are almost the same as if there were no gaps between the strips. On the other hand, the case $h = 2a$ is already an example of a very weak magnetic coupling that result in a current penetration for each strip almost as if the two others were not present.

To further study this point we present in Fig. 3 the field lines calculated for the three arrays of Fig. 2, for applied fields $H_a/H_{\text{pen}} = 0.4$. The left images show the total field and the right ones only the field created by the currents in the superconductor. It is clear that for $h = 0.02a$ and even $h = 0.2a$, the applied field in the space between superconductors is basically shielded by them, in contrast with the case of $h = 2a$, for which the magnetic field is modified near each strip but not enough to make a significant contribution to the other two strips. Another way of seeing this effect is illustrated in the calculations of the field created by currents (right images in each figure). One can observe that in all cases currents create a basically constant field in the spaces between strips. However, when the separation is large the total field lines created by the current are wrapped around each tape, whereas when the separation decreases up to $h/a = 0.02$ the field lines are hardly distinguishable from the case of the three tapes forming a single thicker one.

IV. MAGNETIZATION

In this section we analyze the magnetization of the arrays, calculated from the currents following Eq. (4). The reverse and returning curve can be obtained from the initial one using Eqs. (1) and (2).

There are several important properties of the tapes arrays that can be understood from the magnetization results. In fig. 4 we plot the calculated magnetization M as function of the applied field H_a for a set of three arrays, each with semisides ratio b/a of 0.01 and with different separations $h/a = 0.02, 0.2$, and 2 , respectively. We also plot the magnetization for a single tape with $b/a = 0.01$, another one with $b/a = 0.03$ corresponding to the case that the three tapes are one on top of each other. One can observe that the general trend is that the saturation magnetization remains the same for all cases, whereas the initial slope of the magnetization curve changes. The slope is largest in absolute value for the case of a single thin tape ($b/a = 0.01$), has intermediate values for the three arrays (the largest slope is for the array with large separation and smallest for the one with the smallest one), and finally the case with $b/a = 0.03$, which, interestingly, is hardly distinguishable from the array with the smallest separation. The reason for such an enhancement of the initial slope is the demagnetizing effect associated with the large sample aspect ratio, ac-

cording to which the thinner the sample the larger the initial slope [13,10]. In the arrays for which the separation between the tapes is small, the magnetic coupling increase, in agreement with the discussion in section III, so that the sample is behaving as having a larger thickness, and therefore, less demagnetizing effects and less initial slope. In order to study in more detail this effect, we have included in Fig. 4 the calculated magnetization of a single tape with $b/a = 0.05$; this tape corresponds to the array with $h/a = 0.02$ but as if the gaps between the tapes were filled by superconducting material as well. We can see that this case has the smallest slope, and there are large differences with respect to the case of the array of three tapes with separation $h/a = 0.02$. From these results we can conclude that, provided that the strips are close enough, the behavior of a z array is similar to that of a strip of thickness the sum of the superconducting regions of the z -array, and not the sum of the whole z -array including the gaps.

Another interesting feature to study is the effect of the addition of more strips to the array. We compare in Fig. 5 the calculated $M(H_a)$ curves for arrays with a fix distance ($h = 0.2$) and different number of tapes. We include in the figure the two analytically known limits of one infinitely thin strip [4] and for an infinite set of strips [15]. Results show a practical coincidence between the calculated results for a single tape with finite although small thickness and the results from the analytical formula for very thin strips [4]. With adding more tapes, the initial slope of the magnetization (defined as magnetic moment divided by volume, so independent of the superconductor volume) gets smaller in absolute value. We find that even the case of 25 strips is significantly different from the Mawatari case for an infinite stack, so we can conclude that Mawatari's formula should be valid only for a very large number of tapes. In Fig. 6 we show the magnetization curves $M(H_a)$ for arrays of tapes separated a smaller distance ($h/a = 2$). It can be observed that now the differences in the slope are smaller than in the previous case, because there is less magnetic coupling among the tapes.

V. AC LOSSES

In this section we study the imaginary part of the AC susceptibility, χ'' , calculated form the magnetization loops from Eq. (5). χ'' is directly related to the power losses by Eq. (6).

We present in Fig. 7 the calculated results for χ'' as function of the maximum applied AC field H_a corresponding to the magnetization curves of Fig. 4. All curves show a peak at some value of the applied field amplitude. It can be seen that the peak corresponding to the maximum in χ'' (and therefore a change in slope of the AC losses, since they are proportional to χ'' times H_a^2 as seen in Eq. (6)) is shifting to higher fields and

decreasing in magnitude with decreasing separation distance. The reason for that can be obtained from the analogous shifting in the initial slope of the magnetization shown in Fig. 4. Since the AC losses are related to the area of the $M(H_a)$ curve [Eqs. (5) and (6)] and the magnetization saturation is the same for all cases, the key factor for the losses behavior is the initial slope governed by the demagnetizing effects as discussed in Section IV.

Results in Fig. 7 show that the $\chi''(H_a)$ curve goes as H_a^n for low fields, with n ranging from 1.5 for a strip with $b/a = 0.01$, corresponding to the high h/a limit for an x -array, to 1.3 for a strip with $b/a = 0.03$, being the low h/a limit for arrays. These values lie between the known limiting values for infinite slabs ($n = 1$) [1] and thin strips ($n = 2$) [29]. The corresponding power losses (related to χ'' by Eq. (6); see inset) go therefore as H_a^n , with n ranging from 3.5 to 3.3. For high fields, χ'' goes as H_a^{-1} in all cases.

In Figs. 8 and 9 we calculate the results for χ'' as function of the maximum applied AC field H_a with the goal of studying the effect of adding more tapes in the array. We use the same cases as in Figs. 5 and 6, corresponding to two different separations in the arrays ($h/a = 2$, and 0.2, respectively). Again, there is a close relation between the increase in the initial slope of the magnetization curves and the shifting of the position of the peak in χ'' to higher fields. It is interesting to comment, however, that the formulas provided by Mawatari for the case of an infinite array are not adequate to describe quantitatively the AC losses of a finite array even if the array consists of up to 25 tapes.

It is clear from the calculations presented in this section that in order to reduce the magnetic contribution to the AC losses in a real tape made of superconducting filaments with a given thickness one should increase the coupling of the filaments by decreasing the distance between them (see Fig. 7). Also, the results in Fig. 8 and 9 show that the AC losses decrease with increasing the number of filaments while keeping the distance between them, particularly in the case of a small separation ($h/a = 0.2$, Fig. 9). These results can be understood again as arising from the effect of demagnetizing fields. When the superconducting tapes are separated a small distance, the whole tape is acting basically as a single superconducting tape with a thickness equal to the sum of the thickness of the tapes, so the demagnetizing effects are less, the initial magnetization has a smaller slope (in absolute value) as discussed in Section V, and the area of the hysteresis loops is less for a given maximum applied AC field, so that the AC losses are reduced. On the other hand, when the separation of the tapes gets larger, the magnetic coupling between them is less so that they act more like magnetically decoupled filaments with high aspect ratio and the demagnetizing effects act more, increasing the AC losses. It is interesting to notice that when we talk about the separation h being small one should understand it as small compared with the horizontal dimension $2a$ and not to the strip thickness

2b. Calculations of the same values of h/a for the case $b/a = 0.1$ (instead of $b/a = 0.01$ as in the results presented above), for example, yield qualitatively the same effects for the same h/a values.

VI. CONCLUSIONS

We have presented a numerical model for calculating current penetration and field profiles, and magnetization and AC losses of an array of superconducting tapes. In this work we have analyzed the case of an array of vertically arranged superconducting strips. We have found that the demagnetizing effects have strong influences in the magnetic response of the tapes and the AC losses appearing when an AC field is applied. We find that AC losses are reduced when decreasing the vertical separation between filaments. When the vertical separation is small as compared with the filaments width, then the tape is behaving as a single filament with thickness the sum of the superconducting material. These results could be used as guides for designing actual superconducting tapes. Then, in order to optimize the losses, for filaments with a fixed aspect ratio, it is preferable to have a large number of them separated small distances so that there is a good magnetic coupling between them, as has been already experimentally found [28]. The model can be applied to horizontal and matrix arrangements as is presented in the next paper.

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APPENDIX A: FIELD OF FULL PENETRATION

In this section we present a simple way to analytically calculate the field of full penetration of the array, H_{pen} , defined as the minimum applied field in the initial magnetization curve for which current fills the whole of the superconducting region. The penetration field can be calculated in general as minus the field generated by the current distribution \mathbf{H}_J in the last induced current point, where $\mathbf{H}_J = H_J \hat{\mathbf{z}}$ [10,27]. So, both the current distribution at the penetration field and the last induced current point position \mathbf{r}_m must be known to calculate H_{pen} .

In the geometry of a set of rectangular cross-section strips ordered as a z -stack we can distinguish two different cases, depending on whether the number of strips n_f is odd or even. Although in both cases the current distribution at H_{pen} is evident, it is not so for \mathbf{r}_m . The last

induced current point can be found as where the field generated by the currents is maximum in magnitude, since \mathbf{r}_m is the last point where external field is shielded. For the odd number of strips case, \mathbf{r}_m is simply the position of the center of the central strip, although it is not so easy to determine when the number of strips is even.

For a z -stack of an odd number of strips n_f with dimensions $2a$ and $2b$ in the x and z directions separated a distance h the penetration field is therefore

$$H_{\text{pen}}(a, b, h, n_f) = \frac{J_c}{2\pi} \left[F_1(0, a, b) + 2 \sum_{i=1}^{\frac{n_f-1}{2}} F_1((2b+h)i, a, b) \right] \quad (n_f \text{ odd}), \quad (\text{A1})$$

where $F_1(u, t, d)$ is defined as

$$F_1(u, t, d) = 2t \left\{ \arctan \frac{u+d}{t} - \arctan \frac{u-d}{t} + (u-d) \ln \left[\frac{(u-d)^2}{t^2 + (u-d)^2} \right] + (u+d) \ln \left[\frac{(u+d)^2}{t^2 + (u+d)^2} \right] \right\}. \quad (\text{A2})$$

Eq. (A1) has been derived using the expression for the magnetic field created by a completely penetrated strip with uniform J_c calculated by direct integration of the Biot-Savart law. The case $n_f = 1$ reproduces the known result for the penetration field for a strip [10].

As mentioned above, when a z -stack have an even number of strips we must find the last point where current is induced, at which the self field H_J is maximum in magnitude. Owing to the symmetry of the current fronts in the yz plane, this point will be on the z axis. Thus, only maximization of $H_{J,z}$ along the z axis is needed. Since the minimization of the field $H_{J,z}$ is different for every specific value of n_f , we only report the result for $n_f = 2$, which is the most important case, concerning the magnetic coupling, for an even n_f . Then, the last induced current point will be at a position

$$z_m^2 = \frac{1}{2} \left[-a^2 - h\beta + \sqrt{(a^2 + h\beta)^2 + h\beta(2a^2 + h^2/2 + 2\beta^2 + h\beta)} \right], \quad (\text{A3})$$

being z_m the z component of \mathbf{r}_m and β defined as $\beta = 2b + h/2$. The penetration field for two strips is

$$H_{\text{pen}}(a, b, n_f = 2) = \frac{J_c}{2\pi} [F_1(z_m - b - h/2, a, b) + F_1(z_m + b + h/2, a, b)], \quad (\text{A4})$$

where the function $F_1(u, t, d)$ is defined in Eq. (A2).

APPENDIX B: CALCULATION OF INDUCTANCES

In this appendix we calculate the self and mutual inductances used in Eq. (3). These inductances are calculated for closed circuits of the shape of a pair of identical rectangular infinite prisms of dimensions $2a' \times 2b'$ carrying uniform current density. The prisms are set symmetrically to the zy plane, taking the y axis parallel to the infinite direction. The current of the prism set in the $x \geq 0$ region is taken positive, while it is taken negative for the other.

The self and mutual inductances are calculated from the magnetic energy using the equation [25]

$$M_{ij}I_iI_j = W_{ij} = \int_{x_i-a'}^{x_i+a'} dx \int_{z_i-b'}^{z_i+b'} dz A_{y,j}(x, z) J_i - \int_{-x_i-a'}^{-x_i+a'} dx \int_{z_i-b'}^{z_i+b'} dz A_{y,j}(x, z) J_i, \quad (B1)$$

where M_{ij} is the mutual inductance per unit length of two closed circuits labelled as i and j respectively, I_i and I_j are the current intensity flowing through the circuits, W_{ij} is the magnetic energy of the circuits, (x_i, z_i) is the central position of the prism in the $x \geq 0$ region of the i circuit, a' and b' the dimensions of the prisms in the x and z directions respectively, and $A_{y,j}$ is the y component of the vector potential created by the circuit j taking the gauge $\nabla \cdot \mathbf{A} = 0$.

The vector potential $A_{y,j}$ can be calculated by direct integration leading to

$$A_{y,j}(x, z) = \frac{\mu_0 J_j}{2\pi} [F(x - x_j, z) - F(x + x_j, z)], \quad (B2)$$

where the function $F(u, v)$ is defined as

$$F(u, v) = f(a' - u, b' - v) + f(a' - u, b' + v) + f(a' + u, b' - v) + f(a' + u, b' + v), \quad (B3)$$

defining $f(t, d)$ as

$$f(t, d) = \frac{1}{2} \left[t d \ln(t^2 + d^2) - 3 t d + t^2 \arctan \frac{d}{t} + t^2 \arctan \frac{d}{t} \right]. \quad (B4)$$

Taking into account that the current density in the prisms is uniform, M_{ij} can be deduced integrating Eq. (B1) using Eqs. (B2)-(B4), which yield

$$M_{ij} = \frac{\mu_0}{16\pi a'^2 b'^2} [G(x_j - x_i, y_j - y_i) - G(-x_j - x_i, y_j - y_i)], \quad (B5)$$

where the function $G(u, v)$ is defined as

$$G(u, v) = \sum_{k,l,n,m=1}^2 (-1)^{k+l+n+m} \times g(R(k, n)a' + u, R(l, m)b' + v), \quad (B6)$$

defining $R(i, j) = (-1)^i - (-1)^j$, and the function $g(t, d)$ as

$$g(t, d) = \frac{24}{48} t^2 d^2 - \frac{t^4}{96} - \frac{dt^3}{6} \arctan \frac{d}{t} - \frac{td^3}{6} \arctan \frac{t}{d} + \frac{1}{48} (t^4 + d^4 - 6t^2 d^2) \ln(t^2 + d^2). \quad (B7)$$

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FIG. 1. Sketch of the array of superconducting strips.

FIG. 2. Current profiles for an array of three superconducting tapes of width $2a$ and height $2b$ separated a distance (a) $h/a=2$, (b) $h/a=0.2$, and (c) $h/a=0.02$. The profiles correspond to applied fields $H_a/H_{\text{pen}}=0.2, 0.4, 0.6, 0.8$, and 1, where H_{pen} is the penetration field of the array. For the sake of clarity, the separation, thickness and width of strips are not on scale.

FIG. 3. Field lines corresponding to an applied field $H_a/H_{\text{pen}}=0.4$, where H_{pen} is the penetration field of the z -stack, for the stacks of Fig. 2. Right and left figures correspond to the total and self-field magnetic field lines, respectively. The distances are $d/a = 2$ (a,b), 0.2 (c,d), and 0.02 (e,f).

FIG. 4. Initial magnetization as a function of the applied field for, from left to right: a single tape with $b/a = 0.01$, an array of three tapes with $b/a = 0.01$ separated a distance $h/a = 2$, the same array with a separation distance of $h/a = 0.2$, the same array with a separation distance of $h/a = 0.02$, a single tape with $b/a = 0.03$, and a single tape with $b/a = 0.05$. In the inset complete magnetization loops are plotted for the first and last cases.

FIG. 5. Initial magnetization as a function of the applied field for, from left to right: a single tape with $b/a = 0.01$, the theoretical expression for thin strips (almost overlapped), a set of 3 tapes with $b/a = 0.01$ each, a set of 5 tapes with $b/a = 0.01$ each, a set of 9 tapes with $b/a = 0.01$ each, a set of 25 tapes with $b/a = 0.01$ each, and the expression given by Mawatari (Ref. 15) for an array of infinite number of tapes of dimensions $b/a = 0.01$. The separation distance between the tapes in all cases is $h/a=0.2$.

FIG. 6. Same as Fig. 5, but for a separation $h/a=2$.

FIG. 7. Imaginary part of the AC susceptibility χ'' as a function of the amplitude of the AC field H_a corresponding to the curves of Fig. 4, in the same order (the case of a single tape with $b/a=0.05$ is not plotted in this figure). The corresponding power losses are shown in the inset.

FIG. 8. Imaginary part of the AC susceptibility χ'' as a function of the amplitude of the AC field H_a corresponding to the curves of Fig. 5, in the same order.

FIG. 9. Imaginary part of the AC susceptibility χ'' as a function of the amplitude of the AC field H_a corresponding to the curves of Fig. 6, in the same order.





















